STUDYING AND CAPTURING THE COMPLEXITY OF PRACTICE - THE CASE OF THE 'DANCE OF AGENCY[1].

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In this paper I will explore three contrasting teaching and learning environments, including one in which students engage in a 'dance of agency'. I will move from a consideration of classroom practices to a contention that our work as researchers of mathematics education should pay more attention to teaching practices. Further, that understanding and capturing teaching practices will help researchers to cross traditional and elusive divides between research and practice.

INTRODUCTION

In Geoff Saxe's 1999 PME plenary talk he argued for the importance of studying classroom practices, in order to understand the impact of professional development. Saxe defined practice as 'recurrent socially organized activities that permeate daily life' (1999, 1-25). In this paper I will also focus upon practices, arguing that researchers need to study classroom practices in order to understand relationships between teaching and learning (Cobb, Stephan, McClain & Gravemeijer, 2001). I consider classroom practices to be the recurrent activities and norms that develop in classrooms over time, in which teachers and students engage. Practices such as – interpreting cues in order to answer textbook questions (Boaler, 2002a) – for example, may not always be obvious and may require careful attention, but I contend that such actions are extremely important in shaping student understandings. In addition to focusing upon classroom practices I will spend some time considering the teaching practices – the detailed activities in which teachers engage – that support them. The field of mathematics education has reached a highly developed understanding of effective learning environments without, it seems to me, an accompanying understanding of the teaching practices that are needed to support them. Teaching is itself a complex practice and I will argue that our field needs to develop a greater understanding of its nuances, and that capturing 'records of practice' (Ball & Cohen, 1999, p14) may be an important means by which researchers may cross traditional divides between research and practice.

STUDIES OF PRACTICE

Classroom Practices.

As part of a four-year study in California I am following students through different teaching approaches, in order to understand relationships between teaching and learning[2]. The students are in three high schools, two of which offer a choice between an open ended, applied mathematical approach that combines all areas of mathematics (from here

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[1] Pickering, 1995, p116.

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on referred to as ‘reform’) and a ‘traditional’ approach comprising courses of algebra, then geometry, then advanced algebra - taught using traditional methods of demonstration and practice. Students (and parents) choose which approach they take. My research team[3] and I are monitoring approximately 1000 students as they move through their mathematics courses over four years, observing lessons and administering assessments and questionnaires. Approximately 106 students are following the reform approach, 467 the traditional approach and 517 a conceptual approach, which includes aspects of both reform and traditional teaching (I will not report upon the 'conceptual approach' in this paper). In addition to our large scale monitoring, we are studying one or more focus classes from each approach in each school. In these classes we observe and video lessons, and conduct interviews with the teacher and selected students. The student populations that we are following are ethnically and socio-economically diverse. In this paper I will refer to some of the findings emerging from the two schools that offer students a choice between traditional and reform teaching.

In the ‘traditional approach’ teachers demonstrate mathematical methods that students practice in their exercise books. Students sit individually and work alone and the questions they work on are usually short and closed. Part of our analysis of classrooms involved coding the ways that teachers and students spent time in lessons. This revealed that approximately 21% of the time in traditional classes was spent with teachers talking to the students, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole class format. Approximately 48% of the time students were practicing methods in their books, working individually, and the average time spent on each mathematics problem was 2.5 minutes.

In the ‘reform approach’ teachers give students open-ended problems to work on. The problems come from the “integrated mathematics program” (IMP) a curriculum that poses big unit problems and then a series of shorter activities that help students learn methods to solve the unit problems. Often students are given time to explore ideas that they consider later - for example, students play probability games before discussing probabilistic notions. Students work in groups for the majority of class time. Our coding revealed that in the reform classes teachers talked to the students for approximately 16% of the time, and they questioned students, in whole class format for approximately 32% of the time. Approximately 32% of the time the students worked on problems in groups and the average time students spent on each problem was approximately 6.8 minutes.

One interesting observation from our coding of class time was the increased time that teachers spent questioning the whole class in the reform classes. Whereas the teachers in the traditional classes *gave* students a lot of information, the teachers of the reform approach chose to *draw* information out of students, by presenting problems and asking students questions. There is a common perception that reform-oriented approaches are less 'teacher-centered' but teacher questioning, arguably the most important aspect of teachers' work, was more prevalent in reform classes. Indeed the traditional approaches involved less teacher-student interactions of *any* form as the students spent most of their

[3] The research team includes Karin Brodie, Melissa Gresalfi, Emily Shahan, Megan Staples & Toby White.

time working through textbooks alone. Such observations challenge some of the myths and stereotypes that surround different teaching approaches (Rosen, 2000).

The environments of the traditional and reform classes we studied were completely different and the roles students were required to play as learners varied significantly. In the traditional classes the teacher presented material and the students' main role was to listen carefully to the teacher's words and to reproduce the methods they were shown. As a student reported in interview when asked what it takes to be successful in mathematics:

> M: A big thing for me is, like, paying attention because he'll, like, teach stuff - steps at a time. It'll be like here's a step, here's a step. And, like, if I doze off or, like, don't know what's going on or, like, daydreaming while he's on a step and then he, like, skips to the next step and I'm like, "Wo. How'd he get that answer? Like, where am I? I'm retarded." (…) So paying attention. (Matt, Greendale, algebra)

The students' expected role in the traditional classes was relatively passive and many students positioned themselves as 'received knowers' (Belencky, Clinchy, Goldberger & Tarule, 1986). They believed that their main role was to 'receive' the information teachers presented to them, remembering each demonstrated step. In one of our observations of the traditional classes (in a school in which students had chosen between courses described as "traditional" or "IMP") we approached a student and asked him how he was getting on. He replied "Great! the thing I love about traditional teaching is the teacher tells it to you and you get it". At this point the teacher returned a test to the student, with the grade "F" written on the front. The student looked at the grade and turned back to us, saying "but that's what I hate about traditional teaching, you think you've got it when you haven't". This comment seemed to capture the position of received knowing.

The environments of the reform classes were very different and students were required to undertake contrasting roles. In most lessons students were encouraged to propose ideas and theories, suggest mathematical directions, ask each other questions and generally engage more actively. The students were given more 'agency' and they positioned themselves as more active knowers:

> A: To be successful in IMP you have to be real open minded about things (..) So I just kind of feel that being open-minded and looking at things from different perspectives is the key to being successful.
>
> J: In algebra they give you the rule, and in IMP you have to come up with the rule. It was fun to come up with different things. It taught me how to make a rule, to solve a problem. (Andy & Jack, Greendale, IMP).

Part of our study involves understanding the ways students are positioned in classes and the beliefs and identities they develop as learners, but we are also monitoring the students' achievement. In order to assess students we developed tests that included questions from each approach, assessing the mathematics that all students had met. The tests were checked by the teachers from each approach to ensure that they provided a fair assessment. When students began their different approaches there were no significant differences in their performance on tests (ANOVA, f= 1.07. dfw = 1, p >0.05); the cohorts entering the traditional and reform approaches had reached similar levels of prior attainment. After a year in their different environments the students were tested again. At

this point there were still no significant differences in the students' attainment (ANOVA f= 1.84. dfw = 1, $p > 0.05$) Indeed the similarity of the performance of students may be considered remarkable given the completely different ways in which they worked. This degree of similarity on test scores may appear uninteresting as researchers generally look for differences in attainment when students are taught in different ways, but I contend that these results show something extremely important. The students were not achieving at similar levels because the curriculum approaches were not important, but because significant differences in the attainment of different classes taught by different teachers evened out across the large sample of students - indeed HLM analyses of our data set showed that the teacher was the *only* significant variable in the achievement of students. This result highlighted for us the importance of the particular *practices* in which teachers and students engaged and the insufficiency of research that considers teaching approach by looking at test scores without studying classroom interactions (Lerman, 2000).

Recognizing that teaching and learning environments vary within the same curriculum approach we developed a method that focused more closely on the differences in the environments generated in classrooms. This involved coding the ways in which teachers and students spent time. Our research group observed hundreds of hours of classroom videotape to decide on the appropriate categories for classifying time spent. We reached agreement that the main activities in which teachers and students engaged were: whole class discussion, teacher talking, teacher questioning, individual student work, group work, students off topic, student presenting, and test-taking. We then spent a year repeatedly viewing the tapes to agree upon the nature of the different activities. When over 85% agreement was reached in our coding, every 30-second period of time was categorized for 57 hours of videotape. This exercise was interesting as it showed that teachers who were following traditional or reform approaches spent time in different ways. As I have reported, students in reform classes spent less time being shown methods and more time responding to questions. But the coding also showed that teachers following the same broad curriculum approach, whose environments contrasted significantly, spent time in *similar* ways. Thus they spent similar amounts of time on such activities as presenting methods, asking questions, and having students work in groups. This was particularly interesting to note as we knew from extensive observations that the teaching and learning environments created in the different classes were very different. This coding exercise highlighted something interesting – it showed that important differences in learning opportunities were not captured by such a broad grain size. It was not enough to know how teachers and students spent time. At some levels this is not surprising – most educators know that it is not the *fact* that students work in groups, or listen to the teacher, that is important, it is how they work in groups, what the teacher says and how the students respond. But while this may seem obvious, most debates of teaching and learning occur at a broad level of specificity. Politicians, policy makers, parents, and others engage in fierce debates over whether students should work in groups, use calculators, or listen to lectures (Rosen, 2000). Our data suggests that such debates miss the essence of what constitutes good teaching and learning.

One important message we learned from this research was the importance of the work of *teaching*. It became increasingly clear that an understanding of relationships between teaching and learning in the different classes could only come about through studying the

classroom practices in which teachers and students engaged. In the traditional classes many of the practices we observed were similar to those I found in a study in England, such as procedure repetition and cue-based methods (see Boaler, 2002a,b). In the reform classes the practices were much more varied, which may reflect the greater levels of teacher-student interactions in the reform classes. Our sample of focus classes in which we videotaped lessons and interviewed students in the first year included three classes that were working on the same IMP curriculum. Previous research (including my own) may suggest that knowing that a class is working on open problems is sufficient to suggest responses in student learning. However, the environments generated in the three classes were completely different. This difference allowed us to understand the ways in which particular teaching decisions and actions changed the opportunities created for students, as I shall review briefly now. This greater level of nuance in understanding reform-oriented teaching may be important to our field.

The main difference in the environments of the three reform classes we studied emanated from the structure and guidance that teachers gave the students (see Henningsten & Stein, 1997, for a similar finding). One of the classes was taught by 'Mr Life', a mathematics and science teacher who was extremely popular with students. Mr Life related most of the mathematical ideas to which students were introduced to events in students' lives and his classroom was filled with scientific models, as well as plants, and birds. Mr Life valued student thinking and he encouraged students to use and share different methods, but when he helped students he gave them a lot of structure. For example, in one lesson students were asked to design a 'rug' that could serve as a probability space, to map out the probability that a basketball player would score from 60% of her shots. Students had been using rugs as area models in a number of previous lessons. This task was intended to give the students the opportunity to consider different probability spaces but Mr Life waited only 10 seconds before telling the students that they should draw a 10x10 grid for their rug as that was the easiest way to show a 60% probability. Mr Life provided such structure to help the students and make the tasks more accessible but the effect was usually to reduce the cognitive demand of the tasks. Mr Life also asked students closed questions that led them in particular directions, such as "should we multiply or divide now?" The students in Mr Life's class learned to engage in *structured* problem solving, performing the small activities and methods that he required of them. The students were very happy doing so and they performed well on tests.

The second of the reform classes was taught by ‘Mr Freedom’ - a mathematics teacher who wanted students to engage in open problems and to express themselves creatively. Mr Freedom loved the open curriculum he used, but he seemed to have decided that students would learn to use and apply mathematics if he refrained from providing them with structure. When students asked for help, Mr Freedom would tell them that they should work the answers out on their own, or with other students. This broad degree of freedom resulted in students becoming frustrated and annoyed. It seemed that the students did not have the resources (Engle & Conant, 2002) they needed in order to engage with the open problems and they came to believe that Mr Freedom did not care about them. Mr Freedom 's desire to give them space and opportunity often resulted in disorganized classes with unhappy students and a frustrated teacher:

> A: If he explains to us then I think I am able to understand it more. Sometimes he just tells us 'OK, you do this homework' and we don't even get it.
>
> K: I think when the teacher gets frustrated and he starts to like, he doesn't tell you what he wants you to do, but he thinks you know and he gets all upset. (Anna & Kieran, Hilltop, IMP).

Mr Freedom's students were considerably less happy than Mr Life's and they performed less well on tests.

The third class, taught by 'Ms Conceptual', was different again. Ms Conceptual, like Mr Freedom, wanted students to engage in open-ended problem solving. But she did not refrain from helping students as did Mr Freedom, nor did she structure the problems, as did Mr Life, instead she engaged students in what have been described as a set of 'mathematical practices' (RAND, 2003). A panel of mathematicians and mathematics educators in the US, outlined a list of activities in which successful problem solvers engage. They called these mathematical practices and they included such actions as: exploring, orienting, representing, generalizing, and justifying. Such activities have been considered by other researchers and are sometimes labeled in other ways, as processes or strategies (Schoenfeld, 1985). When students were unsure how to proceed with open problems Ms Conceptual encouraged the students to engage in these practices. For example, she would suggest to students that they *represent* problems they were working on, by drawing a picture or setting out information; she would ask them to *justify* their answers; and she would ask them to *orient* themselves, asking such questions as: 'Let's go back - what are we trying to find?' These encouragements were highly significant in giving students access to the problems without reducing their cognitive demand.

There is a common perception that the authority in reform mathematics classrooms shifts from the teacher to the students. This is partly true, the students in Ms Conceptual's class did have more authority than those in the traditional classes we followed. But another important source of authority in her classroom was the domain of mathematics itself. Ms Conceptual employed an important teaching practice - that of *deflecting* her authority *to* the discipline. When students were working on problems and they asked 'is this correct?' - she rarely said 'yes' or 'no', nor did she simply ask 'what do you think?' instead she would ask questions such as: 'have you tried it with some different numbers?' 'can you draw a diagram?' or 'how is this example related to the last one we saw?'. By encouraging these practices Ms Conceptual was implicitly saying: don't ask me – consider the authority of the discipline – the norms and activities that constitute mathematical work. Those who are opposed to the use of reform teaching methods in the US argue that reform methods are non-mathematical, involving students in what they call "fuzzy" mathematics. They argue that 'anything goes' in reform classrooms and they worry that students are left to invent their own methods with no recourse to the discipline. Anti-reformers have won the semantic high ground by casting all reform teaching as un-mathematical, and traditional teaching as 'mathematically correct'. Yet we have found that the traditional teachers in our study do not invoke the discipline of mathematics as the authority for students to reference; the authorities the students draw upon are teachers and textbooks. This raises questions about the ways students cope when they are out of the classroom and away from the sources of authority upon which they come to depend. It also raises questions about the extent to which the students' work in classrooms *is mathematical*. We consider

classrooms such as Ms Conceptuals to be *more* mathematical, because the teacher positions the discipline of mathematics as the authority from which students should draw.

The IMP 1 class we studied, taught by Ms Conceptual, was a ‘retake’ class of students who had previously failed one or more mathematics classes, but they performed almost as well as Mr Life’s mainstream students at the end of the year, and better than Mr Freedom’s students. The three classes followed the same curriculum approach, but Ms Conceptual's class was the only one in which we witnessed open problem solving. Andrew Pickering (1995), a sociologist of knowledge, studied some of the world’s most important mathematical advances, in order to understand the interplay of knowledge and agency in the production of new conceptual systems. He proposed that mathematical advances require an inter-change of human agency and what he calls the ‘agency of the discipline’ (1995, p116). Pickering studied the work of mathematicians and identified the times at which they used their own agency – in creating initial thoughts and ideas, or by taking established ideas and extending them. He also described the times when they needed to surrender to the ‘agency of the discipline’, when they needed to follow standard procedures of mathematical proof, for example, subjecting their ideas to widely agreed methods of verification. Pickering draws attention to an important interplay that takes place between human and disciplinary agency and refers to this as ‘the dance of agency’ (1995, p116). In Ms Conceptual's class we frequently witnessed students engaged in this ‘dance’ - they were not only required to use their own ideas as in Mr Freedom's class nor did they spend the majority of their time ‘surrendering to the agency of the discipline’, as in Mr Life's; instead they learned to interweave standard methods and procedures with their own thoughts as they adapted and connected different methods.

The following extract is taken from a class discussion in one of Ms Conceptual’s older classes. The class is IMP 4 – the fourth year of the ‘reform’ curriculum – and the students have learned to engage in the ‘dance’ with some fluency. In the lesson from which the extract is drawn the students were asked to find the maximum area of a triangle, with sides of 2 and 3 meters and an enclosed angle that they could choose. The lesson is intended to give students the opportunity to find the areas of different triangles and in doing so, develop the formula area = 1/2 ab sin □. The class develop this formula during a 90 minute period of whole class and small group problem solving. The following exchange comes after the class has worked out the areas of different acute triangles, with enclosed angles of 90 degrees, as well as angles a little over and under 90 (to explore whether 90 degrees gave the maximum area). The class has derived the formula and at the point we join the lesson a student has asked whether the formula they have developed works for obtuse as well as acute triangles. The teacher asks all the students to work on that question in groups, then she calls the class together to hear Ryan’s explanation:

Ms C: Now. Let’s go back to your original question. Your original question was, you see how it works on here (points to acute triangle), but you’re not sure how it works on here (points to obtuse triangle). Now you say you figured it out.

Ryan: no, I I I did the same thing, the only difference in my, in the formula was that instead of a Y sin □, I used a Y sin 180 minus □, because I was using this angle (in acute triangle), and that formula.

Ms C: OK

Ryan: but then I just, I realized that that the sin of like if you're, if θ equals 100, like you order around with that one, with that angle up there, if the sin of 100 would be the same as sin of 80, which is the 180 minus 100 –'cause ninety's the meeting point, and then they it goes down on it. Er ninety's like the uh, the highest.

Ms C: ahhhhhhhhhh! Do you wanna—can you? [*gesturing to the board*] You wanna explain on the graph? Does everyone understand what he's saying?

Class: no

Ms C: [*to Ryan*] and did what? Stand on this side, please. Talk about your original hypothesis, because this is real important what he's talking about with the θ, and the one-eighty minus θ.

Ryan: I'm trying to find a general formula for the, this triangle (obtuse). Because I knew that the triangle used to find the height is right there [in acute] and so I knew that that angle would be different, so to find so I did the same thing, the only difference was for that angle right there, I did um 180 minus theta, because if you know, if you know that angle right there is theta, then you know that the two combined have to be 180, so one eighty minus theta would equal that angle? And then I just used that in a formula, and then it was different and you have to look at the triangle to figure out which formula to use, er ,whatever? But then I tried, but then I realized that the sin, the sin graph goes like this [*drawing the sin graph on the board*] So the sin, the graph of the sin goes like that er whatever?. And uh, that's where the ninety is? And it like, if you were like doing this triangle and say you decided to make theta 100 degrees, then 180 minus theta would have been 80? And eighty is like right around over there, what it equals? The sin of eighty is about right there? And then if you were using this formula right here, the sin of just plain theta? And do 100? And one hundred is on the other side, would be right there? So they still equal the same thing.

Female st: Oh, I see

Female st: so it really doesn't matter?

Female st: so even if it's over ninety, or under it?

Ryan: exactly

This extract is interesting because it shows a student engaged in the 'dance of agency'. Ryan takes the formula that the class has developed: area = 1/2 ab sin θ and extends it by replacing sin θ with sin (180 - θ). This enables him to understand why the formula can be used with an obtuse as well as an acute triangle, and why the area is maximized with a right angled triangle. Ryan engages in a practice of considering a method, applying his own thought and developing a new method that works for other triangles. He is engaged in a 'dance' of disciplinary agency and his own agency. This practice is one that the teacher frequently encourages and it is something to which the students referred when they were interviewed. In interviews we asked students what they do when they encounter new mathematical problems that they cannot immediately solve:

> K: I'd generally just stare at the problem. If I get stuck I just think about it really hard and then just start writing. Usually for everything I just start writing some sort of formula. And if that doesn't work I just adjust it, and keep on adjusting it until it works. (Keith, IMP4)

> B: A lot of times we have to use what we've learned (…) and apply it to what we're doing right now, just to figure out what's going on It's never just, like, given. Like "use this formula to find this answer" You always have to like, change it around somehow. (Benny, IMP4)

These students seem to be describing a dance of agency as they move between the standard methods and procedures they know and the new situations to which they would apply them. They do not only talk about their own ideas, they talk about adapting and extending methods and the interchange between their own ideas and standard mathematical methods. The dance of agency is one of the practices we observe being taught and learned in Ms Conceptual's classroom. It is a complex practice subsuming many smaller practices; it takes a lot of careful teaching and it is not commonly seen.

The differences we have noted between the classroom interactions in the different classes have enabled us to consider the work of the teacher in creating environments that encourage successful problem solving. I have documented some of the emerging results from our study in this paper in order to highlight three points:

1. If we are to understand differences in teaching and learning environments, it is insufficient to describe general approaches, even to describe the different, broad ways in which teachers and students spend time. Understanding the ways that students engage with mathematics requires a focus on classroom practices.

2. One practice we regard to be important in enabling students to work productively with open problems is the 'dance of agency'. This practice does not come about through the simple provision of open problems and requires careful teaching.

3. A critical factor in the production of effective teaching and learning environments is the work of *teaching*. In the United States fierce debates rage around the issue of curriculum; our research suggests that greater attention be given to the work of teaching as it is teachers that make the difference between more and less productive engagement.

In the first part of this paper I have highlighted the importance of studying classroom practices if we are to make progress as a field in our understandings of relationships between teaching and learning. In the remainder of this paper I focus upon *teaching practices* and contend that understanding and capturing teaching practices will help researchers to cross divides between research and practice.

A Focus on Teaching Practices.

As we study more classes, especially those where teachers are encouraging mathematical practices and engagement in a 'dance of agency' we see and appreciate the complexity of the work of teaching. Mathematics educators have set out visions of teaching reforms in the US that are elegant in their rationale, and draw from complex understandings of productive learning environments, but such visions belie the complexity of the changes in teaching they require. Our field seems to have developed an advanced understanding of mathematics learning, through a history of research on learning, without as full an understanding of the teaching that is needed to bring about such learning. But a better understanding of the work involved in creating productive learning environments is probably the clearest way to improve practice, at this time. It is my contention that one of the most useful contributions that research in our field can make in future years is to gather knowledge on the work involved in teaching for understanding - in different countries and situations and for different groups of students.

An understanding of the work involved in teaching for understanding must start with a well developed understanding of the act of teaching itself. There is a widespread public perception that good teachers simply need to know a lot. But teaching is not a knowledge base, it is an action, and teacher knowledge is only useful to the extent that it interacts productively with all of the different variables in teaching. Knowledge of subject, curriculum, or even teaching methods, needs to combine with teachers' own thoughts and ideas as they too engage in something of a conceptual dance. Lee Shulman presented one of the most influential conceptions of teachers' knowledge when he defined pedagogical content knowledge, as 'a special amalgam of content and pedagogy that is uniquely the province of teachers' (1987, p9). But Shulman (2003) has reconceived his original model of knowledge to capture the activity that constitutes teaching. He has recently written:

> 'What's missing in that model? How must the model adapt? First, we need more emphasis on the level of action, and the ways in which the transformation processes are deployed during interactive teaching and how these are related, for example, to the elements of surprise and chance. Second, we need to confront the inadequacy of the individual as the sole unit of analysis and the need to augment the individual model with the critical role of the community of teachers as learners. Third, the absence of any emphasis on affect, motivation or passion, and the critical role of affective scaffolding in the teachers' learning must be repaired. Fourth, we need to invite the likelihood of beginning, not only with text (standing for the subject matter to be taught and learned), but with students, standards, community, general vision or goals, etc'. (Shulman, personal communication, 2003).

Shulman's note speaks to the importance of conceiving knowledge as part of a complex set of interactions, involving action, analysis and affect. Teaching is a complex practice that cannot be dichotomized into knowledge and action. Pickering challenges the duality of different agencies in the development of conceptual advances, arguing that mathematics work takes place at the *intersection* of agencies. Teaching similarly, is as an action that takes place at the intersection of knowledge and thought. Just as mathematics learners need to engage in a dance of agency, so to do teachers. In our history of research different groups have generally failed to find connections between the achievement of students and teacher knowledge. Similarly, large-scale curriculum studies have not found correlations between curriculum approach and test success. The reason that such studies do not find correlations is not because teacher knowledge or curriculum are not important, but because they are mediated by practice (Doyle, 1977) and many of our studies have not taken account of the teaching practices that mediate teacher knowledge and curriculum. Understanding and classifying teaching is extraordinarily difficult for the simple reason that teaching is a *practice* that takes place at the intersection of hundreds of variables that play out differently in every moment (Mason & Spence, 1999), a complexity that has led David Berliner to argue that if educational research is to be conceived as a science then it is probably 'the hardest science of all' (2002, p18). Ball (1993) and Lampert (2001) have offered analyses of their own teaching that document the complexity involved. Other researchers have captured some of the complexity by analyzing the many faceted dimensions of teacher knowledge and the inter-relations of knowledge and belief (Even & Tirosh, 1995). I highlight the complexity in this paper in order to consider teacher learning and ways to help research impact practice.

It is well known that much of the research in mathematics education has limited impact on practice. As journal articles accumulate understandings of mathematics teaching and learning, schools and teachers continue relatively unchanged (Tyack & Cuban, 1995). Part of addressing this problem may involve a greater understanding of teaching as a complex act of reasoning, or a dance. In the past educators have communicated general principles that teachers have not found useful in their teaching. But just as students need to learn by engaging in problems, not only by reading solutions in books, so do teachers. The educational research communicated to teachers via journals may be the equivalent of teaching students mathematics by giving them pages of elegantly worked problems - they may learn from such work, but it is unlikely. Dancers could not learn their craft by observing dance, or reading about successful dance. Teachers too need to learn their 'dance' by engaging in the practice of teaching and our field may need to address this fact in the ways we communicate findings from research.

One well recognized issue related to the usefulness of educational research is that of *medium* - teachers do not read educational journals and so the production of scholarly articles, filled with technical, academic jargon is not useful. Another, less recognized issue is that of *grain size*. It seems that many educational findings are at an inappropriate grain size for teachers, who do not need to know, for example that group work is effective, they need to know how to make it work - how to encourage student discussion and how to reduce or eliminate status differences between students (Cohen & Lotan, 1997). In studying the learning environment of Ms Conceptual's class – one in which students collectively solve problems, building on each others' ideas in a stream of high level problem solving, we have realized that she enacts a teaching practice that is both critical and highly unusual. One of the results of the teaching reforms in the US is a large number of teachers who now ask students to present their ideas to the rest of the class. In our observations of other classes we have only ever seen teachers ask students to present *finished* solutions. Ms Conceptual, in contrast, asks students to present ideas before anyone has finished working on the problems. I would identify this as a particular teacher practice that Ms Conceptual employs, that has important implications for the learning environment that ensues. One impact of this teacher practice is that the students need to build on each others' ideas as the problem solving act happens collectively. Another important shift is the role that the 'audience' is required to play when students are presenting. In other, more typical classes students present *finished* problems and the majority of the students watching have already attained the same answer. The role of the audience in watching the presentation is relatively passive and many students appear bored and not to be listening. When students are presenting ideas at the board in Ms Conceptual's class, the rest of the students are highly attentive because they have not finished the problem and they want to see the ideas communicated and join in with the problem solving. This particular teaching practice – asking students to present before they are finished – has important implications for learning yet it is a fine-grained practice. Recommendations from research often remain at a larger grain size – suggesting, for example, *that* presentations take place; it seems that more detailed conceptions of the act of teaching may be needed if we are to understand and impact practice.

A third issue, related to medium is the *form of knowledge* produced. Paul Black (2003) makes an important point - teachers do not simply take research knowledge and apply it

in their classrooms, they need to *transform* knowledge into actions. Basil Bernstein (1996) also highlights the transformation that takes place when discourse 'moves', arguing for the existence of a 'recontextualising principle' which 'selectively appropriates, relocates, refocuses and relates other discourse to constitute its own order' (p33). Ball and Cohen (1999) provide a new vision of teacher learning and professional development that addresses the transformation and recontextualisation required. They suggest that teacher learning be situated in strategically documented *records of practice* - 'copies of student work, video-tapes of classroom lessons, curriculum materials and teachers' notes all would be candidates' (p14). Ball and Cohen (1999) write that:

> 'Using artifacts and records of practice, teachers have opportunities to pursue questions and puzzles that are deeply rooted in practice, but not of their own classrooms (…) Three features stand out about such a curriculum for professional education. One, that it centers professional inquiry in practice. Using real artifacts, records, moments, events and tasks permits a kind of study and analysis that is impossible in the abstract. Second it opens up comparative perspectives on practice. (…) Third it contributes to collective professional inquiry.' (p24).

Ball and Cohen's vision concerns the creation of new forms of professional development in which teachers learn in and through practice. One message I take from this work is the increased power that educational research can exert if researchers transform their findings into records of practice. As our study of teaching and learning in three high schools evolves, we are developing ways of communicating the results of the research not only through journals but through videos of teaching[4] and portfolios of student work. This does not mean simply communicating findings but creating opportunities for teachers to conduct their own inquiries within records of practice. In one presentation to a large audience we showed a tape of Ms Conceptual's students collaboratively problem solving. Mr Freedom happened to be in the audience and he reported afterwards that he was stunned by the videotape. He told us that he was totally enthralled to see a teacher teaching the same curriculum and achieving greater levels of student engagement. He watched it with huge interest, noting the teacher's practices, which he immediately tried to emulate when he returned to his classroom. Mr Freedom's attempt at generating the collaborative problem solving achieved in Ms Conceptual's class was not totally successful, unsurprisingly, as the success of her teaching rests upon carefully and slowly established classroom norms, but this event did illustrate for us the potential of such records for impacting practice. Records of practice that researchers could produce would not convey results, or even principles, in clear and unambiguous terms, instead they would present some of the complexity of classroom experience in order to provide sites for teachers' own inquiry, reasoning and learning. If we are to make the years of research on students' mathematics learning have an impact then it seems we need to find newer and more effective ways to communicate practices and the creation of records of practice may encourage this.

CONCLUSIONS

We have reached an important time in our field, when groups of researchers are looking not only to develop theories, but to impact practice. In the United States a committee of

[4] Such as the video example that will be shown in my talk.

educational researchers, convened by the National Academy of Education (2000), urged those funding research to prioritize studies designed to impact practice, with findings that would 'travel'. The group coined the term 'travel' to move beyond the idea of 'transferring' knowledge. This conception may be insufficient – I contend, as has Black (2003), that knowledge needs not only to travel but to be *transformed*. Researchers can leave the transformation process to teachers by providing principles that teachers first must envision as practices and then convert to actions as part of the teaching dance. Alternatively researchers may produce artifacts that encourage a special kind of analysis - grounded in practice. It seems that researchers, as well as professional developers, can aid the process of transformation by *capturing* some of the practices of teaching and converting them into a set of carefully documented records of practice. I have communicated in this paper my own conviction that researchers in mathematics education need to study the practices of classrooms in order to understand relationships between teaching and learning and they need to capture the practices of classrooms in order to cross divides between research and practice. Our field needs to puzzle over the ways that research knowledge may be transformed into student learning (Wilson & Berne, 1999) and it is my belief that greater attention to the complexity of teaching practices may serve this transformation.

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